can be obtained exactly from Eqs. (10) and (12) by setting $\eta = 0$ or from Eq. (16) with $a_0 = \frac{1}{2}$ since $E_3(0) = \frac{1}{2}$. The result is

$$n_{e0} = (2\pi\nu_1^3/uc^2)(1/S)(1 + 2/S + 2/S^2)\exp(-S) \quad (17)$$

n_{e0} is seen to be independent of the values of atomic constants P and n and depends only on the frequency ν_1 at the edge of the primary continuum. This result is physically obvious for the steady, infinite shock case since all of the photons for $\nu > \nu_1$ are absorbed, independent of the magnitude or form of the spectral absorption coefficient. The exact values of P and n are required only for the subsequent decay of the electron number density. This prediction that the electron number density at shock front due to photoionization being dependent only on ν_1 can be checked if measurements are made on different atomic gases. We attempted to compare the theoretical results with Holmes and Weymann's measurements in Ref. 2 but the comparisons are of doubtful value in view of several uncertainties.[7]

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Base Pressure Correlation in Supersonic Flow

M. W. Su* and J. M. Wu†
The University of Tennessee Space Institute
Tullahoma, Tenn.

BASE drag can be a significant fraction of the total drag of a body flying at supersonic speeds. The estimation of the base pressure has received a great deal of attention. However, because of the complicated and coupled nature of the near wake, no satisfactory correlation parameter for accurately predicting the base pressure for flow with transition in the wake has yet been found. In this note, a correlation parameter is identified and proposed, which is derived on the basis of the reduced Reynolds number in the mixing region.

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* Graduate Research Assistant; now Senior Engineer, Boeing Company, Space Division, Huntsville, Ala. Member AIAA.

† Associate Professor of Aerospace Engineering, Member AIAA.

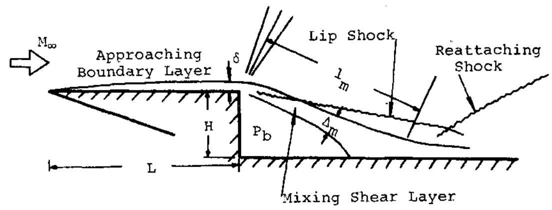

Fig. 1 Sketch of base flow field.

In a locally separated region, the usually defined Reynolds number based on the characteristic length of a body is rather ambiguous and does not yield a proper physical meaning. To a base flow (see Fig. 1), one may easily sense that the characteristic length for such type of flow is the total mixing shear length l_m and the mixing width Δ_m at near the recompression region. An attempt was made to correlate the base pressure to these two lengths. A very encouraging base pressure correlation was obtained. The only difficulty is that the l_m and the Δ_m lengths are not known a priori. Therefore, one must find a way to relate these two lengths to the known body lengths; namely, the model length upstream of a two-dimensional rearward facing step L and the step height H. Physically, the length L is related to the boundary-layer thickness δ prior to separation which will influence the mixing width Δ_m and the step height H which is related to the mixing shear length l_m.

The ratio of inertia to viscous forces in the mixing shear layer, or the physically correct reduced Reynolds number Re_c in this region, may be expressed as,

$$Re_c = \rho_\infty U_\infty (U_\infty / l_m)/\mu_\infty (U_\infty/\Delta_m^2) = (\rho_\infty U_\infty l_m/\mu_\infty)(\Delta_m/l_m)^2 \quad (1)$$

where l_m and Δ_m are taken as the characteristic dimensions of the mixing region as discussed. The estimation of l_m and Δ_m is often difficult. If one assumes that

$$l_m \sim H \quad (2)$$

$$\frac{\Delta_m}{\delta} \sim \left(\frac{l_m}{\delta}\right)^n \sim \left(\frac{H}{\delta}\right)^{n(Re_{\infty,L}, M_\infty)} \quad (2a)$$

where the exponent n is a function of freestream Reynolds number based on L and the freestream Mach number M_∞. The exponent n may depend strongly on whether the mixing shear layer is laminar, transitional or turbulent. Based on (2) and (2a), the reduced Reynolds number Re_c of Eq. (1)

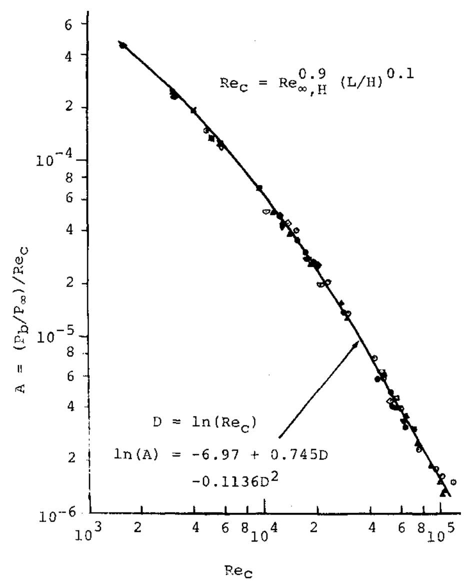

Fig. 2 Base pressure correlation for rearward facing step.

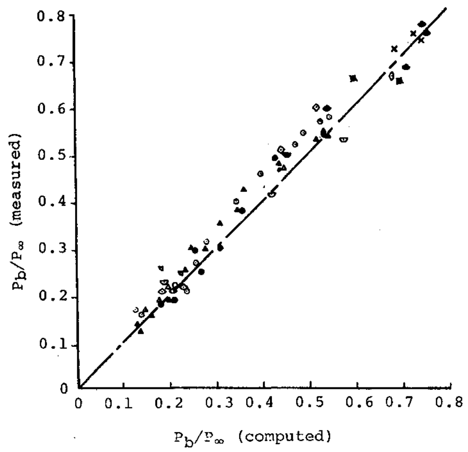

Fig. 3 Comparison of measured base pressure and that computed from correlation.

Table 1 Data used in Figures 2 and 3

Symbol	M_∞	H, in.	L/H	Investigators
⊙	2.50	0.7500	5.33	Wu and Su[3]
△	3.50	0.7500	5.33	Wu and Su[3]
●	2.50	0.4400	9.03	Smith[4]
▲	3.50	0.4400	9.03	Smith[4]
⬢	2	0.0100	20	Chapman[5]
⬡	2	0.1000	20	Chapman[5]
×	2.25	0.0072	31.40	Rom[6]
⊗	3.55	0.0072	31.40	Rom[6]
⬯	2.25	0.0165	13.20	Rom[6]
⬮	3.55	0.0165	13.20	Rom[6]
◇	2.25	0.0256	13	Rom[6]
◆	3.55	0.0256	13	Rom[6]
◡	2.25	0.0335	7.40	Rom[6]
▾	3.55	0.0335	7.40	Rom[6]
▽	2.25	0.0790	8	Rom[6]
▼	3.50	0.0790	8	Rom[6]
⬠	2.25	0.0790	15	Rom[6]
◈	3.50	0.0790	15	Rom[6]

becomes,

$$Re_c = \frac{\rho_\infty U_\infty H}{\mu_\infty}\left(\frac{H}{\delta}\right)^{2n-2} \tag{3}$$

If one also neglects the dependence of δ on Mach number, then,

$$\frac{\rho_\infty U_\infty H}{\mu_\infty} = \left(\frac{\rho_\infty U_\infty L}{\mu_\infty}\right)\left(\frac{H}{L}\right) \sim \left(\frac{H}{\delta}\right)^m \left(\frac{L}{H}\right)^m \left(\frac{L}{H}\right)^{-1} \tag{4}$$

and

$$Re_c = (H/\delta)^{m+2n-2}(L/H)^{m-1} \tag{5}$$

where for very small pressure gradient flow, approximately, m is 2 or 5 depending on whether the boundary layer on the body is laminar or turbulent, respectively. The reduced Reynolds number is thus related to δ, H and L, which are the important parameters of the base flow. This reduced Reynolds number is very likely to be the correlation parameter for the base pressure.

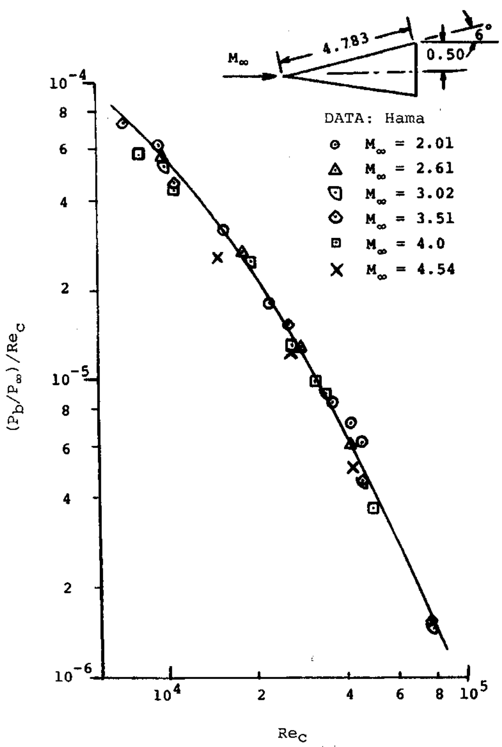

Fig. 4 Base pressure correlation for a six degree wedge.

Based on their mixing theory, Crocco and Lees[1] were able to show the important trends in the variation of base pressure with Reynolds number, L/H and the nature of boundary layer. They argued that the base pressure is a function of the mixing rate and δ. The mixing rate and δ depend not only on Reynolds number but also on the location of transition (whether on the body or on the separated region). For an intermediate wake transition region, the rate of upstream movement of transition in the wake with increasing Reynolds number is governed largely by the parameter L/H. For fully laminar or turbulent flow, since L/H is not as important, the base pressure ratio depends mainly on the ratio δ/H as correlated by Chapman.[2] For transitional wake flow, $m = 2$ and if one assumes $n = 0.9$, the correlation parameter reads

$$Re_c \sim (H/\delta)^{1.8}(L/H) \sim (Re_{\infty,H})^{0.9}(L/H)^{0.1} \tag{5a}$$

Because of the contrast in dependence of the base pressure on the mixing rate and the boundary-layer thickness, as pointed out by Crocco and Lees, it is proposed to normalize the base pressure ratio by the same correlation parameter. The correlation is shown in Fig. 2 for data[3-6] (given in Table 1) in the range: $0.05 \times 10^6 < Re_{\infty,L} < 1.8 \times 10^6$, $2.0 < M_\infty < 3.55$, $5.33 < L/H < 31.4$, $0.0072\text{ in.} < H < 0.75\text{ in.}$

The proposed base pressure correlation for a two dimensional rearward facing step is,

$$P_b/P_\infty = Re_c e(-6.97 + 0.745D - 0.1136D^2) \tag{6}$$

where

$$D = \ln(Re_c)$$

The comparison of measured base pressure ratio and the base pressure ratio computed by Eq. 6 is shown in Fig. 3, which indicates that the correlation is indeed a reasonable one.

Similar correlation for a 6 degree wedge[7] is shown in Fig. 4. Note that for both Figs. 2 and 4, the body boundary layer is laminar before separation, and that this correlation parameter applies to fully laminar flow as well.

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Use of Padé Fractions in the Calculation of Blunt Body Flows

ANDREW H. VAN TUYL*
U. S. Naval Ordnance Laboratory, Silver Spring, Md.

IN Ref. 1, a method involving Padé fractions is given for calculation of the axially symmetric flow of a perfect gas past a blunt body of revolution. Terms of the Taylor expansion of the stream function in the neighborhood of the nose of the shock up to and including degree 8 are used, and are calculated by means of explicit expressions due to Lin and Shen.[2] This Taylor expansion does not converge at the body,[3] and the use of Padé fractions has been found to be a means of obtaining convergence.[4] A related method, using continued fractions, has been given by Moran.[5] Other methods for removing the divergence of series expansions in the blunt body problem are given in Refs. 6–8.

In the present Note, the method of Ref. 1 is generalized to include calculation of an arbitrary number of terms of the Taylor expansion of the stream function. Calculations are carried out in several cases using the present method together with a method of characteristics program written for the IBM 7090 by Thompson and Furey.[9]

A uniform flow of a perfect gas is assumed ahead of the bow shock, with Mach number M_∞ and ratio of specific heats γ. Let x and r be cylindrical coordinates with origin at the nose of the body, and let the shock be given by

$$(r/R_s)^2 = 2(x + x_0)/R_s - B_s(x + x_0)^2/R_s^2 + \dots \quad (1)$$

where R_s is the radius of curvature of the shock nose, B_s is the shock bluntness, and x_0 is the shock detachment distance. The equation of the body which would produce the shock is

$$(r/R_b)^2 = 2(x/R_b) - B_b(x/R_b)^2 + \dots \quad (2)$$

where R_b is the radius of curvature of the nose, and B_b is the bluntness.

In the inverse problem, in which the shock is given and the body which would produce it is calculated, we first find coefficients of the Taylor expansion of the stream function in cylindrical coordinates. Bernoulli's equation and the vorticity equation are used as the starting point of this calculation, as in the procedure of Ref. 2. Subroutines for power series manipulations are used, based in part on algorithms due to Leavitt.[10] As in Ref. 1, a transformation is then made to coordinates τ and ξ such that the shock is the coordinate surface $\tau = 0$. The Taylor expansion of the stream function in

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* Research Mathematician. Member AIAA.

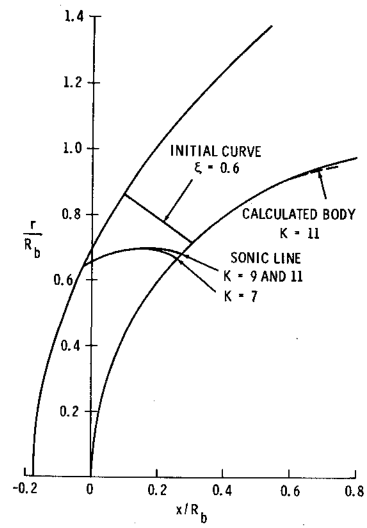

Fig. 1 Sonic line and initial curve in the flow past a sphere at $M_\infty = 4$.

the new coordinates will be written in the form

$$\frac{\psi}{\rho_\infty q_\infty R_s^2} = \sum_{i=1}^{\infty} \psi_i(\tau)\xi^{2i} \quad (3)$$

where ρ_∞ and q_∞ are the density and velocity magnitude, respectively, of the uniform flow, and

$$\psi_i(\tau) = \sum_{j=0}^{\infty} \psi_{ij}\tau^j \quad (4)$$

When K-1 terms of the series for the shock equation are used, the series for $\psi_i(\tau)$ is known through the term of degree $2K$-$2i$.

Padé fractions[11] with numerator and denominator of equal or nearly equal degrees are formed from the power series for the $\psi_i(\tau)$ and certain of their derivatives, and are used in place of the power series in the calculation of the flow. These sequences of Padé fractions are found to converge for values of τ on the body, while the power series from which they are obtained are divergent.

These approximations are used to obtain coefficients of the equation of the body, together with values of x_0/R_s and R_b/R_s. Coefficients of the expansions for the pressure, density, and velocity magnitude on the body in powers of s/R_b are then obtained, where s is the arc length measured from the nose. These expansions have the forms shown in Ref. 1, Sec. 9. Each coefficient of these expansions and of Eq. (2) can be expressed in terms of a finite number of the $\psi_i(\tau)$ and their derivatives evaluated for $\tau = \tau_0$, where τ_0 is the smallest positive root of the equation $\psi_1(\tau) = 0$. The number of coefficients of each series which can be calculated with sufficient accuracy is about half the number of terms used in Eq. (1). Finally, the partial sums are replaced by Padé fractions, and the latter are used in the flow calculations. Examples show that these rational approximations are more accurate than the partial sums near the sonic point.

Flow quantities in the shock layer are expanded in powers of $\cos\alpha$, where as defined in Ref. 1, α at a given point is the angle between the tangent to the curve τ = constant through that point and the positive x axis. Padé fractions are then formed from these expansions, and are used to calculate the flow. In particular, when 9 terms of the series for the pressure are used, we obtain a rational approximation of the form

$$p \simeq \sum_{i=0}^{4} A_i(\tau)\cos^{2i}\alpha \Big/ \sum_{i=0}^{4} B_i(\tau)\cos^{2i}\alpha \quad (5)$$

It follows from a property of Padé fractions that the rational approximations for u, p, ρ, q^2, and c^2 obtained in this way are exact at the shock in the inverse problem when more than three terms of the power series are used,[1] where u is the x